

**The S parameter in a technicolour model
with explicit chiral symmetry breaking**

E. Pallante*

I.N.F.N., Laboratori Nazionali di Frascati, Via E. Fermi, 00144 Frascati ITALY

Abstract

We derive the S parameter of the electroweak radiative corrections in a scaled-QCD technicolour and using a Nambu-Jona Lasinio model for hadronic low energy interactions. In this framework deviations from low energy QCD can be quantitatively deduced. We induce explicit chiral symmetry breaking in the NJL model through the current-quarks mass term. It is shown that the prediction for the S parameter can be sensitively reduced respect to the chiral prediction.

ROM2F/94/32
hep-ph/9408232

* email: pallante@vaxtov.roma2.infn.it
fax: ++39-6-9403-427

The S parameter of the electroweak radiative corrections is defined in terms of two of the gauge bosons vacuum polarization functions as [1]

$$\alpha S \equiv 4e^2[\Pi'_{33}(0) - \Pi'_{3Q}(0)], \quad (1)$$

where α is the electromagnetic fine structure constant, the index 3 refers to the isospin $SU(2)_L$ current, while the index Q refers to the $U(1)_{em}$ current. The primed functions arise from the expansion in q^2 of the Π functions defined as

$$i \int d^4x e^{-iqx} \langle J_X^\mu(x) J_Y^\nu(0) \rangle \equiv -g^{\mu\nu} \Pi_{XY}(q^2) + q^\mu q^\nu \text{terms}, \quad (2)$$

with $XY = 33, 3Q$. By neglecting terms of order higher than q^2 one has:

$$\begin{aligned} \Pi_{33}(q^2) &\sim \Pi_{33}(0) + q^2 \Pi'_{33}(0), \\ \Pi_{3Q}(q^2) &\sim q^2 \Pi'_{3Q}(0), \end{aligned} \quad (3)$$

where $\Pi_{3Q}(0) = 0$ because of the QED Ward identity.

In a technicolour model where isospin and parity are conserved the S parameter can be reexpressed in terms of the isospin-vector and axial vector two point Green's functions through the relations

$$\Pi_{33} = \frac{1}{4}(\Pi_{VV} + \Pi_{AA}), \quad \Pi_{3Q} = \frac{1}{2}\Pi_{VV}, \quad (4)$$

where Π_{VV} and Π_{AA} are defined according to eq. (2) with $XY = VV, AA$.

From eq. (1) and relations (4) the S parameter can be written in terms of the Vector and Axial two-point functions as:

$$S = -4\pi(\Pi'_{VV}(0) - \Pi'_{AA}(0)), \quad (5)$$

where $\Pi'_{XY}(0) = d/dq^2 \Pi_{XY}(q^2)|_{q^2=0}$.

The two-point vector and axial functions can be explicitly derived in effective quark models *à la* Nambu-Jona Lasinio (for a review see also [2, 3]) formulated for hadronic low energy interactions [4, 5, 6]. They have been calculated in the ENJL model with $n_f = 2, 3$ and in the chiral limit in [5], while a calculation in the non chiral limit with a different from the present approach can be found in [7]. They can be written in the usual VMD form with scale dependent meson parameters and $Q^2 = -q^2$:

$$\begin{aligned}
\frac{1}{-Q^2}\Pi_{VV}(Q^2) &= -\frac{f_V^2(Q^2)M_V^2(Q^2)}{M_V^2(Q^2)+Q^2} \\
\frac{1}{-Q^2}\Pi_{AA}(Q^2) &= -\frac{f_\pi^2(Q^2)}{Q^2} - \frac{f_A^2(Q^2)M_A^2(Q^2)}{M_A^2(Q^2)+Q^2},
\end{aligned} \tag{6}$$

where in QCD $f_\pi = 93.3$ MeV is the pion decay constant, f_V and f_A are the couplings of the vector meson to the external vector current and of the axial meson to the external axial current, M_V and M_A are the masses of the vector and axial mesons.

By substituting $\Pi'_{VV}(0)$ and $\Pi'_{AA}(0)$ in the definition of the S parameter we obtain:

$$S = 4\pi[f_V^2(0) - f_A^2(0)]. \tag{7}$$

Using the effective action approach the values at $Q^2 = 0$ of f_V^2 and f_A^2 in the chiral limit are [4]:

$$\begin{aligned}
f_V^2(0) &= \frac{N_c}{16\pi^2} \frac{2}{3} \Gamma\left(0, \frac{M_Q^2}{\Lambda_\chi^2}\right) \\
f_A^2(0) &= \frac{N_c}{16\pi^2} \frac{2}{3} g_A^2(0) \left[\Gamma\left(0, \frac{M_Q^2}{\Lambda_\chi^2}\right) - \Gamma\left(1, \frac{M_Q^2}{\Lambda_\chi^2}\right) \right].
\end{aligned} \tag{8}$$

The expression for S results:

$$S = 4\pi \left[\frac{N_c}{16\pi^2} \frac{2}{3} \Gamma\left(0, \frac{M_Q^2}{\Lambda_\chi^2}\right) (1 - g_A^2(0)) + \frac{N_c}{16\pi^2} \frac{2}{3} g_A^2 \Gamma\left(1, \frac{M_Q^2}{\Lambda_\chi^2}\right) \right]. \tag{9}$$

The g_A parameter is the mixing parameter between axial and pseudoscalar mesons given by:

$$g_A(0) = \frac{1}{1 + 4G_V \frac{M_Q^2}{\Lambda_\chi^2} \Gamma\left(0, \frac{M_Q^2}{\Lambda_\chi^2}\right)}, \tag{10}$$

with G_V the four-quark vector coupling constant and the Γ functions are defined as

$$\Gamma(n-2, \epsilon) = \int_\epsilon^\infty dz \frac{1}{z} e^{-z} z^{n-2}. \tag{11}$$

The function $\Gamma(0, \epsilon) = -\ln \epsilon - \gamma_E + \mathcal{O}(\epsilon)$ corresponds to the divergent contribution in a momentum-cutoff regularization scheme, while $\Gamma(1, \epsilon)$ gives the first finite contribution.

The S parameter is a function of the adimensional ratio M_Q^2/Λ_χ^2 , where M_Q is the infrared cutoff of the effective techni-meson theory and Λ_χ is the ultraviolet cutoff of the effective techni-meson theory. By assuming a technicolour model which is a scaled QCD model (i.e. by assuming the same adimensional ratio M_Q^2/Λ_χ^2 for both QCD and technicolour theories) we will use the numerical low energy QCD values for M_Q and Λ_χ to get a prediction for the S parameter.

By inserting the numerical values $\Lambda_\chi = 1.160$ GeV, $M_Q = 0.265$ GeV and $g_A(0) = 0.61$, with $G_V = 1.263$, valid in the chiral limit [4] we obtain

$$S = N_c \cdot (0.1 + 0.02). \quad (12)$$

The first term, which is the logarithmically divergent term, gives the bulk of the contribution, while a small correction comes from the second term which is finite. The value of eq. (12) is in good agreement with the estimation by Peskin and Takeuchi [1] obtained from the experimental data by using low energy QCD dispersion relations and rescaling to technicolour energies.

The present parametrization gives the possibility to study a strong dynamics which is like the low energy QCD dynamics, but which can include deviations in the mass spectrum.

In what follows we induce explicit chiral symmetry breaking ($E_\chi SB$) in the ENJL model with the addition of the current-quark mass term and study its effect on the S parameter.

By assuming a technicolour spectrum with N_d isodoublets of technifermions (\tilde{u}, \tilde{d}) two different pictures can be analysed:

I) $m_{\tilde{u}} = m_{\tilde{d}} \neq 0$, for each isodoublet; the mass matrix is given by N_d (2×2) degenerate subblocks and isospin is conserved.

II) $m_{\tilde{u}} \neq m_{\tilde{d}}$ in each isodoublet; the mass matrix is non degenerate and isospin symmetry is broken.

We focus on item *I*), while item *II*) is under study.

1 S in the non chiral limit

The NJL effective fermion Lagrangian is

$$\mathcal{L} = \bar{q}(\hat{D} - \mathcal{M})q + \frac{8\pi^2 G_S}{N_c \Lambda_\chi^2} (\bar{q}^a q^b)(\bar{q}^b q^a) - \frac{8\pi^2 G_V}{N_c \Lambda_\chi^2} [(\bar{q}_L^a \gamma_\mu q_L^b)(\bar{q}_L^b \gamma_\mu q_L^a) + L \rightarrow R], \quad (13)$$

where we have introduced the explicit χ SB current-quark mass term; the non-renormalizable four-fermion scalar and vector interactions are generated by the integration over the high frequency modes of quarks and gluons. $G_S(\Lambda_\chi)$ and $G_V(\Lambda_\chi)$ are non perturbative coupling constants and Λ_χ is the ultraviolet cutoff of the effective theory. Bosonization of the Lagrangian (13) introduces scalar, pseudoscalar, vector and axial meson degrees of freedom: the integration over quarks degrees of freedom gives rise to the effective meson Lagrangian whose parameters are generated by one quark-loop calculation. To derive the S parameter in the presence of $E\chi$ SB mass term one has a priori to take into account two effects: the solution M_i of the mass-gap equation, which is the pole of the constituent-quark propagator, becomes a function of the current-quark mass m_i where i is the flavour index. After the bosonization corrections to the vector and axial meson parameters can be expressed in terms of the masses M_i .

1.1 The Mass-Gap equation

The mass-gap equation given by the Lagrangian (13) defines the dressed current-quark propagator as the sum of the bare current-quark propagator with mass m_i and the tadpole contribution generated by the four-quark interaction with coupling G_S . The mass of the dressed current-quark propagator is then

$$M_i = m_i - \frac{1}{2} \left(\frac{8\pi^2 G_S}{N_c \Lambda_\chi^2} \right) < \bar{q}_i q_i > \quad (14)$$

where $< \bar{q}_i q_i >$ is given by

$$\begin{aligned} < \bar{q}_i q_i > &= -i N_c \int \frac{d^4 p}{(2\pi)^4} \frac{M_i}{p^2 - M_i^2} \\ &= -\frac{N_c}{16\pi^2} 4M_i^3 \Gamma(-1, \frac{M_i^2}{\Lambda_\chi^2}). \end{aligned} \quad (15)$$

The graph of figure 1 shows the solution $M_i(m_i)$ rescaled for technicolour as a function of the coupling G_S for different values of the current-quark masses m_i , while the graph of figure 2 shows the solution $M_i(m_i)$ for $G_S = 1.216$ which is the value obtained from the best fit of ref. [4] in the chiral limit. Assuming for the techni-pion decay constant the value $f_\pi^T = 250$ GeV [1], in a QCD like effective technicolour theory we define the ultraviolet cutoff $\Lambda_T \simeq 4\pi f_\pi^T \simeq 3$ TeV. The values

of both masses M_i and m_i in the graphs can be thought as QCD values rescaled by the relation $M_T = M_{QCD} \cdot \Lambda_T / \Lambda_\chi$ with $\Lambda_\chi = 1.160$ GeV.

M_i is a sensitively increasing function of the current-quark mass m_i . The perturbative expansion in powers of the current-quark masses has a limited validity range.

1.2 The calculation

To calculate the non chiral corrections to the f_V and f_A parameters which enter S we use the effective action approach, already used in the chiral limit in ref. [4].

The bosonization of the Lagrangian (13) and the transformation from current-quarks to constituent-quarks

$$Q_L = \xi q_L \quad Q_R = \xi^\dagger q_R, \quad (16)$$

with $\xi = \sqrt{U} = \exp(2i/f_\pi \Phi)$ the square root of the usual exponential representation of the pseudoscalar meson octet Φ , leads to the effective low energy action for physical mesons after the integration over quarks and gluons degrees of freedom:

$$e^{i\Gamma_{eff}[H, W_\mu^\pm; v, a, s, p]} = \int \mathcal{D}G_\mu e^{i \int d^4x -\frac{1}{4}G_{\mu\nu}^2} \int \mathcal{D}\bar{Q}\mathcal{D}Q e^{-i \int d^4x \bar{Q} D_E Q}. \quad (17)$$

H, W_μ^\pm on the right-end side are the scalar, vector and axial meson fields which are the *auxiliary* fields of the bosonized Lagrangian (13) and the effective action is calculated in the presence of external sources v, a, s, p .

The fermionic euclidean operator in the constituent-quark base is:

$$D_E = \gamma_\mu \nabla_\mu - \frac{1}{2}(\Sigma - \gamma_5 \Delta) - H(x). \quad (18)$$

The covariant derivative ∇_μ

$$\nabla_\mu = \partial_\mu + iG_\mu + \Gamma_\mu - \frac{i}{2}\gamma_5(\xi_\mu - W_\mu^-) - \frac{i}{2}W_\mu^+ \quad (19)$$

contains the vector and axial meson fields W_μ^+ and W_μ^- and the vector and axial-vector currents

$$\begin{aligned} \Gamma_\mu &= \frac{1}{2}\{\xi^\dagger[\partial_\mu - i(v_\mu + a_\mu)]\xi + \xi[\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger\} \\ \xi_\mu &= i\{\xi^\dagger[\partial_\mu - i(v_\mu + a_\mu)]\xi - \xi[\partial_\mu - i(v_\mu - a_\mu)]\xi^\dagger\}. \end{aligned} \quad (20)$$

The scalar fields Σ and Δ are proportional to the current quark mass matrix \mathcal{M}

$$\begin{aligned}\Sigma &= \xi^\dagger \mathcal{M} \xi^\dagger + \xi \mathcal{M}^\dagger \xi \\ \Delta &= \xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi.\end{aligned}\tag{21}$$

The scalar field $H(x)$ acquires a non-zero vacuum expectation value (VEV) which is responsible of the spontaneous chiral symmetry breaking in the chiral limit: $H(x) = \langle H \rangle + \sigma(x)$ where the fluctuation $\sigma(x)$ defines the true scalar meson field. Its VEV is the solution of the equation

$$\frac{N_c \Lambda_\chi^2}{8\pi^2 G_S} 2 \langle H \rangle_i + \langle \bar{Q}_i Q_i \rangle = 0,\tag{22}$$

which arises from imposing the extremum condition of the effective action (17) in the absence of external sources and vector fields, i.e. in the mean field approximation:

$$\left. \frac{\delta \Gamma_{eff}}{\delta H} \right|_{H=\langle H \rangle, \xi=1, W_\mu^\pm=0, s=p=v=a=0} = 0.\tag{23}$$

In the mean field approximation (which implies $\xi = 1$) the VEV of the scalar density of the constituent-quarks $\langle \bar{Q}_i Q_i \rangle$ coincides with $\langle \bar{q}_i q_i \rangle$ of the current-quarks. Then eq. (22) implies that the VEV of the scalar field is related to the masses $M_i(m_i)$ solutions of the mass-gap equation through the scalar density $\langle \bar{q}_i q_i \rangle$. In the presence of current-quark masses m_i the identity $M_i = m_i + \langle H \rangle_i$ is implied for each flavour i .

The effective low energy action (i.e. for $Q^2 < \Lambda_T^2$) for techni-meson fields is given by the determinant of the fermionic operator D_E . Because we are interested in the real part of it we can compute the determinant of the squared operator

$$\Gamma_{eff} = -\frac{1}{2} \ln \det(D_E^\dagger D_E)_\epsilon\tag{24}$$

regularized in some scheme. The squared operator $D_E^\dagger D_E$ is a bosonic differential operator bounded below by the constituent quark masses squared \tilde{M}_i^2 :

$$D_E^\dagger D_E \equiv -\nabla_\mu \nabla_\mu + \tilde{M}_i^2 + E,\tag{25}$$

where, referring to eq. (18), $\tilde{M}_i \mathbf{1} = 1/2 \Sigma + \langle H \rangle = \mathcal{M} + \langle H \rangle$ with $\xi = 1$ in first approximation. This shows that the constituent-quark masses \tilde{M}_i coincide with the current-quark masses solution of the mass gap equation. From now on $\tilde{M}_i \equiv M_i$.

In the loop-expansion approach the effective action is given by the perturbative series around the free part of the operator $D_E^\dagger D_E$: $D_E^\dagger D_E \equiv (D_E^\dagger D_E)_0 + \delta = -\partial_\mu \partial^\mu + M_i^2 + \delta$. This defines the constituent free quark propagator $-\partial_\mu \partial^\mu + M_i^2$ and the perturbation δ .

In ref. [4] the fermionic determinant has been calculated in the chiral limit using the heat-kernel expansion in the proper-time regularization scheme (for details on this technique see [8]). This method has the advantage to preserve gauge covariance at each step of the expansion.

Consistency with the loop-expansion approach requires that the infrared cutoff of the heat-kernel expansion of the fermionic squared operator is the constituent quark mass matrix $M_i \mathbf{1}$, both in the chiral and non chiral limit. In the first case M_i coincides with the VEV of the scalar field: $M_i \mathbf{1} = \langle H \rangle_i \mathbf{1} = M_Q \mathbf{1}$, where M_Q appeared in the calculation (9) of the S parameter in the chiral limit.

In the non chiral limit a simple result follows: the low energy meson parameters f_V and f_A are those calculated in the chiral limit with the constituent-quark mass M_Q replaced by its non chiral value $M_i = \langle H \rangle_i (M_i) + m_i$, where $\langle H \rangle$ is in turn a function of M_i , solution of the mass-gap equation for each flavour i . The S parameter of eq. (9) follows with the same substitution and its numerical value corresponds to the contribution of one technicolour isodoublet (\tilde{u}, \tilde{d}) in the isospin limit.

Figure 3 shows the parameter S/N_c as a function of the constituent-quark mass M_i in the case of one isodoublet (\tilde{u}, \tilde{d}) with degenerate masses. It is noticeable that the maximum value of S is for the ratio M_i/Λ_T , or equivalently M_i/Λ_χ , which approximately corresponds to the chiral limit in low energy QCD, while its numerical value decreases rather sensitively by increasing the current techni-quark masses.

Non-perturbative values of the current-techni-quark masses can push the S parameter towards zero.

In the case that all masses of the isodoublets in the model are equal the formula (9) is easily generalized to n_f flavours by multiplying by $n_f/2 = n_d$ with n_d the number of isodoublets in the model. Although no extra-information is given on the techni-particles spectrum the present parametrization reproduces in a clear form the effect of explicitly chiral symmetry breaking.

$q\bar{q}$ states with current masses $m_i > \Lambda_T$ cannot be treated in this framework. Alternative Heavy-quark-effective-theory as an expansion in inverse powers of the mass m_i could be used. Although, from the present behaviour one can infer that

the higher mass of the state the lower is its contribution to the S parameter.

If $E\chi$ SB terms are relevant, higher dimensional effective fermion interactions, proportional to the current techni-quark masses and suppressed by inverse powers of the ultraviolet cutoff, can modify the leading behaviour of the NJL model. The relevance of higher dimensional operators proportional to powers of momenta Q^2 has been studied in [6]. The presence of higher dimensional terms can extend the range of validity of the perturbative expansion in powers of the current-quark masses and further reduce the numerical value of the S parameter.

Acknowledgements

I am grateful to Roberto Petronzio for having called my attention to this problem and for many stimulating and useful discussions.

FIGURE CAPTIONS

- 1) The solution of the mass-gap equation M_i in TeV as a function of the scalar coupling constant G_S for given values of the current techni-quark masses $m_i=0, 0.3, 0.5, 0.8, 1.1$ TeV. The values of M_i and m_i can be thought as QCD values rescaled by the relation $M_T = M_{QCD} \cdot \Lambda_T/\Lambda_\chi$. In particular the values of the techni-quark masses m_i in figure correspond to QCD values $m_i = 0, 0.1, 0.2, 0.3, 0.4$ GeV.
- 2) The solution of the mass-gap equation M_i as a function of the current-quark masses m_i for a given value of $G_S = 1.216$ given by the fit of ref. [4] in the chiral limit.
- 3) The parameter S/N_c as a function of the solution of the mass-gap equation M_i . The maximum is approximately at the value of M_i obtained in the chiral limit.

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